H.S 2^{nd} Year

Question Bank (Chapter wise)

From 2015—2019.

Chapter 1 *Relation and Function* Marks(5)

2019.

1. Let $A = \{x: 1 < x < 10, x \text{ is an odd natural number}\}$ and $B = \{y: 90 < y < 100, y \text{ is a prime number}\}$. Write the number of relations from A to B. 1

2. Let $f: R \to R$ is defined by $f(x) = 3x - 2$ and $g: R \to R$ is defined by $g(x) = \frac{x+2}{3}$. Show that $f.g = g.f$ 4

2018

1. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$. 1
2. Let L be the set of all lines in XY plane and R be the relation in L defined as

   $R = \{(l_1, l_2): l_1 \text{ is parallel to } l_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$. (OR) 4

3. Show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of the function $f: [-1, 1] \to Range\ f$. 4

2017

1. If the function $f = \{(1,5), (2,6), (3,4)\}$ from the set $A = \{1, 2, 3\}$ to the set B is invertible, find the set B. 1
2. Let $f: R \to R$ and $g: R \to R$ be two functions defined by $f(x) = |x|$ and $g(x) = [x]$, where [x] denotes the greatest integer less than or equal to x. Find $(fog)(5.75) \text{ and } (gof)(-\sqrt{5})$. 2+2
3. Show that the relation R in **R**, defined by $R = \{(a, b): a \leq b\}$ is reflexive and transitive, but not symmetric. 4

2016

1. Let $A = \{1, 2, 3\}$. For x, y ϵ A, let xRy if and only if $x > y$. Write down R as a subset of AxA. 1

2. What is the domain of the function $f(x) = \frac{1}{x-2}$? 1
3. Show that the relation R in the set Z of integers given by, $R = \{(x, y): 5 \; divides \; x - y\}$ is an equivalence relation. Find the set of all elements related to 0. 3+1
4. A function $f: R \to R$ is defined by $f(x) = 4x^3 + 5, \; x \in R$. Examine if f is one-one and onto. 2+2

2015

1. If $A = \{0, 1, 3\}$, what is the number of relations on A? 1
2. A function $f: R \to R$ is defined by $f(x) = 2x^2$. Is the function f one-one, and onto? Justify your answer. 2+2
3. Let L be the set of all lines in the xy plane and R be the relation in L defined by $R = \{(l_i, l_j): l_i \; parallel \; to l_j, for \; all \; i, j\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 7x + 5$. 3+1

Chapter 2 *Inverse Trigonometric Function* Marks(5)

2019

1. Write down the range of $f(x) = cot^{-1}x$. 1
2. Let the mapping $f(x) = ax + b$, $a > 0$, maps [-1, 1] onto [0, 2]; show that $cot(cot^{-1}7 + cot^{-1}8 + cot^{-1}18) = f(2)$. 4
3. Find the value of $cos^{1}x + cos^{-1}\left\{\frac{1}{2}\left(x + \sqrt{3}\sqrt{1 - x^2}\right)\right\}$, $\frac{1}{2} \leq x \leq 1$.

2018

1. What is the domain of the function $cosec^{-1}$? 1
2. Prove that $tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x, -\frac{1}{\sqrt{2}} \leq x \leq 1$ 4
3. Show that $sin^{-1}\frac{12}{13} + cos^{-1}\frac{4}{5} + tan^{-1}\frac{63}{13} = \pi$. 4

2017

1. Find the value of x such that $cos(sin^{-1}x) = \frac{1}{2}$. 1

2. Prove that $cot^{-1}\left(\frac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}\right) = \frac{x}{2}, where\ \alpha = sinx,\ x\epsilon(0, \pi/4)$. 4

3. Prove that $2tan^{-1}\left[\sqrt{\left(\frac{a-b}{a+b}\right)}tan\frac{\theta}{2}\right] = cos^{-1}\left(\frac{acos\theta+b}{a+bcos\theta}\right)$. 4

2016

1. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \pi$, prove that $x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} = 2xyz$ 4

2. Prove that $tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{7} + tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. 4

2015

1. Find the principal value of $sin^{-1}\left(sin\frac{3\pi}{5}\right)$. 1
2. If $tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) = \alpha,\ prove\ that\ x^2 = sin2\alpha$. 4

Chapter 3, 4 *Matrices and Determinants* Marks(13)

2019

1. Find all positive values of 2x2 determinats whose entries are from the set $\{-1, 0, 1\}$. 1
2. Let A be askew-symmetric matrix of odd order. Write the value of $|A|$. 1
3. Let A be a matrix of order 3, such that $|A| = -9$. Find the value of $|-3A^{-1}|$. 1
4. Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ 4

OR

5. Without expanding show that $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 2 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 2 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 2 \end{vmatrix} = 0$ 4
6. If $A = \begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$, then show that $I + A = (I - A)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ 6

OR

7. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, then find A^{-1}. And hence solve the system of equations:

$x + 2y - 3z = -4, \quad 2x + 3y + 2z = 2, \quad 3x - 3y - 4z = 11$ 6

2018

1. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$. 1
2. Let |A|=k. If B is the matrix obtained by interchanging two rows of A then |B|=? 1
3. Find $adjA$ when $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.
4. Express the following matrix as a sum of a symmetric and skew-symmetric matrix.

$$\begin{matrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{matrix}$$

OR

5. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 1+2\text{n} & -4\text{n} \\ \text{n} & 1-2\text{n} \end{bmatrix}$ where n is any positive integer. 4

6. Solve the following system of equations by matrix method: 6

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

OR

7. Show that $\begin{vmatrix} x & x^2 & y+z \\ y & y^2 & z+x \\ z & z^2 & x+y \end{vmatrix} = (y-z)(z-x)(x-y)(x+y+z)$

2017

1. If $A = \begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix}$ and $A + A^T = \text{I}$, write down the general value of α. 1
2. If A is a matrix of order 3x4 and B is a matrix of order 4x5, what is the order of the matrix $(AB)^T$**?** 1
3. Let A be a 3x3 determinant and |A|=7. Find the value of |2A|. 1
4. Show that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$. 4

   OR

5. Without expanding the determinant at any stage, show that

$$\begin{vmatrix} x^2 & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B$$

   Where A and B are determinants of order 3x3 not involving x.

6. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11, \quad 3x + 2y - 4z = -5, \quad x + y - 2z = -3$ 6

   OR

7. If $f(x) = x^3 - 6x^2 + 9x - 4$, find $f(A)$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

2016

1. If $(5 \quad x \quad 1)\begin{pmatrix}4\\2\\7\end{pmatrix} = (35)$, find x. 1
2. For what value of x, the matrix $\begin{bmatrix}2-x & 3\\-5 & 1\end{bmatrix}$ is not invertible? 1
3. If A is a square matrix of order 3 such that $|A| = 5$, then find $|A.adjA|$. 1
4. If $A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}. 2+2=4

   OR
5. Using elementary row operation, find the inverse of the matrix $\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}$ 4
6. Prove that $\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$ 6

   OR
7. Show that $\begin{vmatrix}1+x & 1 & 1\\1 & 1+y & 1\\1 & 1 & 1+z\end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right), x \neq 0,\ y \neq 0,\ z \neq 0.$

   6

2015

1. If $[5 \quad 6 \quad 7]A = [13 \quad 23]$, what is the order of the matrix A? 1
2. If A is a nonsingular matrix such that $A^2 + A - I = 0$, what is A^{-1} ? 1
3. What is the co-factor of 7 in the determinant $\begin{vmatrix}4 & 5 & 6\\5 & 6 & 7\\13 & 15 & 17\end{vmatrix}$? 1
4. If $A = \begin{bmatrix}0 & -1 & 2\\2 & -2 & 0\end{bmatrix}$ and $B = \begin{bmatrix}0 & 1\\1 & 0\\1 & 1\end{bmatrix}$, find a matrix C such that $CAB = I = ABC$, where I is the 2x2 unit matrix. 4

   OR
5. Using elementary row operation, find the inverse of the matrix $\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$. 4
6. If x, y, z are all different and $\begin{vmatrix}x & x^2 & 1+x^3\\y & y^2 & 1+y^3\\z & z^2 & 1+z^3\end{vmatrix} = 0$, prove that $xyz = -1$ 6

OR

7. If $a \neq p;\ b \neq q;\ c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0,$ then find the value of $\frac{a}{p-a} + \frac{\mathrm{q}}{\mathrm{q-b}} + \frac{\mathrm{r}}{\mathrm{r-c}}$.

Chapter 5, 6 *Continuity and differentiability, Application of derivative* Marks(15)

2019

1. If $2^x = 3^y$, then find $\frac{dy}{dx}$. 1
2. Write the interval in which the function $f(x) = cosx$ is strictly increasing. 1
3. Show that the function defined by $f(x) = \left|1 - x + |x|\right|, x \epsilon R$ is a continuous function. 4
   OR
4. Discuss the applicability of Rolle's theorem to the function $f(x) = x^2 + 1$ on [-2, 2].
5. If $y = \sqrt{e^{\sqrt{x}}}, find\ \frac{dy}{dx}$. 4
6. If $y = \frac{1}{2} cos^{-1}\left(\frac{1-4x^3}{1+4x^3}\right), x \geq 0,$ find $\frac{dy}{dx}$. 4

   OR

7. Determine the set of all points where the function $f(x) = x\,|x|$ is differentiable.
8. If $y = 3cos(\log x) + 4\,sin(\log x), show\ that\ x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. 4
9. Find the maximum and minimum values of the function
   $f(x) = x + \sin 2x\ \ on\ \ [0, 2\pi]$ 6
10. Prove that the area of a right angled triangle of a given hypotenuse is maximum when the triangle is isosceles. 6

2018

1. Which one of the following is true? For the real function $f(x) = \begin{cases} x + 2 & if\ x \leq 1 \\ x - 2 & if\ x > 1 \end{cases}$

   (i). f is continuous at all real numbers $x > 1\ and\ x < 1$.

   (ii). f is continuous at all real numbers $x \geq 1$.

   (iii). F si continuous at all real numbers $x \leq 1$.

   (iv). F is continuous at $x = 1$.

2. If $x^{2/3} + y^{2/3} = a^{2/3}, find\ \frac{d^2y}{dx^2}$. 4

   OR

3. If $y = (tan^{-1}x)^2$, show that $(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} - 2 = 0$.

4. State Mean value theorem and verify it for the following function: $f(x) = x^2 \ for \ x \in [2, 4]$. 4

OR

5. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$.
6. Find the intervals in which the function $f(x) = \sin 3x, \ \ x \in \left[0, \frac{\pi}{2}\right]$ is (i) increasing (ii). decreasing. 3+3=6

OR

7. Find the maximum and minimum values, if any, of the following function:
$$f(x) = \sin x - \cos x, \ \ 0 < x < 2\pi.$$

2017

1. Write down the range of $f(x) = cosec^{-1}x$. 1
2. Show that the function defined by $f(x) = x - [x]$ is discontinuous at all integral points, where $[x]$ denotes the greatest integer less than or equal to x. 4

OR

3. If $y = e^{acos^{-1}x}, -1 \le x \le 1$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.
4. If $y^x + x^y = 1$, find $\frac{dy}{dx}$. 4

OR

5. State Mean value theorem and verify it for the following function: $f(x) = \ x^2 - 4x - 3$, $x \in [1, 4]$. 1+3=4
6. Find the equation of the tangents to the curve $y = \cos(x + y), -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$. 6

OR

7. Find the local maximum and local minimum value, if any, of the following functions:

   (i). $f(x) = \sin x + \cos x, \ \ 0 < x < \pi/2$.

   (ii). $g(x) = \frac{x}{2} + \frac{2}{x}, \ \ x > 0$.

2016

1. A function $f: R \to R$ is defined as follows $f(x) = \begin{cases} x & if \ x \le 1 \\ 5 & if \ x > 1 \end{cases}$. Which one of the following is true ? 1

(i). f is continuous at 0 and 1.

(ii). f is continuous at 1 and 2.

(iii). f is continuous at 0 and 2.

(iv). F is continuous at 0, 1 and 2.

2. Which onr of the following is true? For the function $f(x) = \cos x$, 1

   (i). f is strictly decreasing in $(\pi, 2\pi)$

   (ii). f is strictly increasing in $(\pi, 2\pi)$

   (iii). f is neither increasing nor decreasing in $(\pi, 2\pi)$

3. Show that $f(x) = |x - 3|$ is continuous but not differentiable at x=3. 2+2=4

   OR

4. If $y = (\sin x)^{\log x}$, find $\frac{dy}{dx}$. 4
5. Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its slope 2/3. Also find the equation of the tangent at that point. 2+2=4

   OR

6. State Rolles's theorem and verify it for the following function. $f(x) = x^2 + 2x - 8,\ x \in [-4, 2]$. 2+2=4
7. Find the maximum and minimum values, if any, of the following function. 3+3=6

   (i). $f(x) = 3x^4 + 4x^3 - 12x^2 + 12,\ x \in R.$

   (ii). $g(x) = -x^2 \log x,\ x \in R.$ OR

8. Find the intervals in which the function $f(x) = \frac{4x^2+1}{x}, x \neq 0$, is (i). increasing (ii). decreasing.

2015

1. Is the derivative of an even function even? 1
2. Is the function $f(x) = x^2,\ x \in R$ increasing? 1
3. If $y + \sin y = \cos x$ then find the values of y for which $\frac{dy}{dx}$ is valid? 4
4. If a function is differentiable at a point, prove that it is continuous at that point. 4

   OR

5. Using Rolle's Theorem, find at what points on the curve $y = \cos x - 1 \ in \ [0, \ 2\pi]$ the tangent is parallel to x-axis.
6. Find the maximum and minimum value of the following functions; if exist. 3+3=6

   (i). $f(x) = \frac{x^2-x+1}{x^2+x+1}; \quad x \in R$ (ii). $f(x) = \log x, \quad x > 0.$

Chapter 7, 8 *Integration, Application of Integration* Marks(20)

2019

1. Evaluate $\int 2xf^{/}(x^2)dx$. 1
2. Evaluate $\int \frac{1}{x}\left(\frac{1-x}{\sqrt{1-x^2}}\right) dx$ 4

   OR
3. Evaluate $\int \frac{\cos 8x+1}{\tan 2x-\cot 2x} dx$.
4. Evaluate $\int_0^1 \frac{3-x^2}{(3+x^2)^2} dx$. 4

   OR

5. Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$
6. Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ 6

2018

1. Evaluate the following integrals: (i) $\int x(\log x)^2\, dx$ OR $\int \frac{x\, dx}{(x-1)^2(x+2)}$ 4
2. (ii). $\int_0^{\pi/2} \log sinx\, dx$ OR $\int_{-1}^{2} |x^3 - x|\, dx$ 4
3. Evaluate $\int_1^4 (x^2 - x)dx$ as the limit of a sum. 6
4. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and $x-axis$. 6

   OR

5. Find the area of the region enclosed between the two circles: $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

2017

1. Evaluate the following integrals: (i). $\int \frac{\sin 2x}{a^2 sin^2 x + b^2 cos^2 x} dx,\ a \neq b$ OR $\int e^x cos\, xdx$
2. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx,\ a > 0$. Hence evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x cos\, x} dx$ 4

   OR
3. Evaluate: $\int_0^{\pi} \frac{xsin\, x}{1+cos^2 x} dx$ 4
4. Evaluate $\int_0^4 (x + e^{2x})dx$ as the limit of sum. 6

5. Find the area lying above the x-axis and included between the curves $x^2 + y^2 = 8x$ and $y^2 = 4x$. 6

OR

6. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4. y = 0$ and $y = 4$ into three equal parts. 6

2016

1. Evaluate the following integrals: (i). $\int (sin^{-1}x)^2\, dx$ OR $\int \frac{dx}{(x+1)(x+2)}$ (ii). $\int_0^{\pi/4} log(1 + \tan x)dx$ OR $\int_0^{\pi} \frac{x - \sin x}{1+cos^2x} dx.$ 4+4=8
2. Find the area of the region in the first quadrant enclosed by x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32.$ 6

OR

3. Find the area of the region common to the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$
4. Evaluate $\int_0^2 (x^2 + 1)dx$ as a limit of sum. 6

2015

1. Evaluate any one of the integrals:

(i). $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$ (ii). $\int \sqrt{x^2 - a^2}\, dx$ 4

2. Prove that $\int_{-a}^{a} f(x)dx = 0$, when f is an odd function. Hence evaluate $\int_{-1}^{1} log\left(\frac{2-x}{2+x}\right) dx.$ 4

3. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis. 6

4. Evaluate : $\int_0^1 tan^{-1} \frac{2x-1}{1+x-x^2} dx$ OR 6

5. Evaluate $\int_1^3 (x^2 + x)dx$ as the limit of a sum.

6. Find the area bounded by $y = x^2$ and $y = |x|$ 6

OR

7. Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$.

Chapter 9 *Differential Equation* Marks(9)

2019

1. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} - 7\left(\frac{dy}{dx}\right)^3 + 6y = 0.$ 1
2. Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ 4
3. Form the differential equation satisfied by $(x-a)^2 + (y-b)^2 = r^2$, where a and b are arbitrary constant. 6

2018

1. What are the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$ 1
2. Form the differential equation of the family of circles touching the y-axis at origin. 4

   OR

3. Solve the differential equation: $sec^2x\ tany\ dx + sec^2y\ tanx\ dy = 0.$
4. Solve the differential equation: $(xdy - ydx)\ ysin\left(\frac{y}{x}\right) = (ydx + xdy)xcos\left(\frac{y}{x}\right).$ 4

2017

1. Find the order and degree of the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
2. Solve: $(1-x^2)\frac{dy}{dx} - xy = 1$ 4

   OR

3. $(x^2 + xy)dy = (x^2 + y^2)dx.$
4. Solve the differential Equation: $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0.$ 4

2016

1. Solve: $\frac{dy}{dx} + y = x$ 4

   OR

2. Form a differential equation by eliminating the arbitrary constants 'a' and 'b' from $y = e^x(a\cos x + bsin\ x)$

3. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$. 4

2015

1. Solve (any one) 4

   (i). $(1 + x^2)\frac{dy}{dx} + y = tan^{-1}x$ (ii). $x\frac{dy}{dx} = y - xtan\left(\frac{y}{x}\right)$.

2. Find the equation of a curve passing through the origin, given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the co-ordinates of the point. 4

Chapter 10, 11 *Vector and Three Dimension* Marks(17)

2019

1. Write the equation of the plane passing through (a, b, c) and parallel to xy-plane. 1
2. If $\vec{a} = 6\hat{\imath} + 8\hat{\jmath}$ and $\vec{b} = 3\hat{\jmath} + 4\hat{k}$ then determine the vector component of $\vec{a}$ along $\vec{b}$.

   OR

3. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$. 4
4. Find the shortest distance between the lines $\hat{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and $\hat{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$. 6

   OR

5. Find the equations of two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at $\frac{\pi}{3}$.
6. Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$. Hence find the area of the parallelogram whose diagonals are the vectors $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$. 6

   OR

7. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\hat{r}.(\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 5$ and $\hat{r}.(3\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$.

2018

1. Find the unit vector in the direction of the vector $\vec{a} + \vec{b}$, where $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$. 1
2. What is the vector equation of the line passing through the points (-1, 0, 2) and (3, 4, 6).
3. What are the direction cosines of the normal to the plane Z=2. 1
4. Find the area of the triangle with vertices (1, 1, 2), (2, 3, 5) and (1, 5, 5). 4

   OR

5. Prove that $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$, if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq 0, \vec{b} \neq 0$.
6. Find the shortest distance between the lines $\hat{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\hat{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$. 4

OR

7. Find the direction cosines of the unit vector perpendicular to the plane $\hat{r}.(6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}) + 1 = 0$ and passing through the origin.
8. Find the vector equation of the plane passing through the intersection of the planes $\hat{r}.(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 7$, $\hat{r}.(2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}) = 9$ and the point (2, 1, 3). 6

OR

9. Find the equation of the plane which contains the line of intersection of the planes $\hat{r}.(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0$, $\hat{r}.(2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ and is perpendicular to the plane $\hat{r}.(5\hat{\imath} + 23 - 6\hat{k}) + 8 = 0$.

2017

1. If $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{b}$=$3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}$ are such that $\vec{a} \perp \vec{b}$, what is the value of λ ?
2. What is the equation of the plane passing through (α, β, γ) and parallel to the plane $x + y + z = 0$. 1
3. If the planes $2x - 4y + 3z = 5$ *and* $x + 2y + \lambda z = 12$ are perpendicular to each other, what is the value of λ ? 1
4. Express the vector $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b}$=$3\hat{\imath} + \hat{k}$ and the other is perpendicular to $\vec{b}$. 4

OR

5. Show that the vector of magnitude $\sqrt{51}$ which makes equal angles with the vectors $\vec{a} = \frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$, $\vec{b}$=$\frac{1}{5}(-4\hat{\imath} - 3\hat{k})$ and $\vec{c} = \hat{\jmath}$ is $-5\hat{\imath} + \hat{\jmath} + 5\hat{k}$. 4
6. Find the length and foot of the perpendicular from the point $(1, 1, 2)$ to the plane $\hat{r}.(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + 5 = 0$. 4

OR

7. Find the equation of the line through the point (-1, 2, 3) which is perpendicular to the lines $\frac{x}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and $\frac{x+3}{-1} = \frac{y+2}{2} = \frac{z-1}{3}$. 4
8. Find the Cartesian as well as the vector equation of the planes passing through the intersection of the planes $\hat{r}.(2\hat{\imath} + 6\hat{\jmath}) + 12 = 0$ and $\hat{r}.(3\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 0$, which are at unit distance from the origin. 6

OR

9. Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$. 6

2016

1. What is the unit vector along the vector $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$? 1
2. What is the direction cosines of the normal to the plane $3x + 2y - 3z = 8$? 1
3. What is the equation of xy plane? 1
4. Find the area of the parallelogram whose diagonals are given by the vertices $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$. 4

   OR

5. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, show that $\vec{a}$ and $\vec{b}$ are perpendicular to each other. 4
6. Find the shortest distance between the lines whose vector equations are given by $\hat{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(-2\hat{\imath} + \hat{\jmath} + 3\hat{k})$ and $\hat{r} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$. 4

   OR

7. Find the acute angle between the planes whose vector equations are $\hat{r}.(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 5$ and $\hat{r}.(3\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = 3$. 4
8. Find the vector equation of the plane passing through the intersection of the planes $\hat{r}.(2\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 2$ and $\hat{r}.(3\hat{\imath} + \hat{\jmath} - 2\hat{k}) = -2$ and perpendicular to the vector $\frac{1}{a^2}$.
   6

   OR

9. Find the equation of the plane that makes intercepts a, b and c on x, y and z axes respectively. Also, if p is the length of the normal from the origin to this plane, prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$. 3+3

2015

1. What are the direction cosines of the vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$? 1
2. If the distance of a plane from the origin be 'd' and direction cosines of the normal to the plane through origin be (l, m, n), what are the co-ordinates of the foot of the normal ?
3. What are the equation of the planes parallel to xz-plane and at a distance 'a' from it?
4. Using vectors prove that angle in a semicircle is a right angle. 4
   OR
5. Using vectors prove that $cos(\alpha - \beta) = cos\alpha\, cos\beta + sin\alpha\, sin\beta$.
6. Find the vector equation of a plane in normal form. 4

   OR

7. Find the equation of a plane passing through a given point and perpendicular to a given vector in vector form.
8. Show that the lines $\hat{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda(3\hat{\imath} - \hat{\jmath})$ and $\hat{r} = \left(4\hat{\imath} - \hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{k}\right)$ are coplanar. Also, find the equation of the plane containing both these lines. 6

Chapter 12 *LPP* Marks(6)

2019

1. Solve the linear programming problem graphically. 6

   Maximize $z = 20x + 15y$, subject to the conditiona

   $2x + y \leq 200,$

   $x + y \leq 150 \quad and \quad x \geq 0, \quad y \geq 0.$

OR

3. Maximize and Minimize
   $z = 5x + 2y$, subject to the conditions,

   $x - 2y \leq 2$,
   $3x + 2y \leq 12$,
   $-3x + 2y \leq 3$ and $x \geq 0, \quad y \geq 0.$

2018

1. Solve the linear programming problem graphically 6
   Maximize and Minimize $z = 6x + 3y$
   Subject to $4x + y \geq 80$
   $x + 5y \geq 115,$
   $3x + 2y \leq 150,$
   $x \geq 0, \quad y \geq 0.$

   OR

2. Maximize and Minimize $z = 800x + 1200y$
   Subject to $3x + 4y \leq 60,$
   $x + 3y \leq 30,$
   and $x \geq 0, \quad y \geq 0.$

2017

1. Solve the linear programming problem graphically 6
   Maximize and Minimize $z = 5x + 3y$
   Subject to $3x + 5y \leq 15,$
   $5x + 2y \leq 10,$
   and $x \geq 0, \quad y \geq 0.$

OR

2. Maximize and Minimize $z = 3x + 9y$
   Subject to $x + 3y \leq 60,$
   $x + y \geq 10,$
   $x \leq y,$
   and $x \geq 0, \quad y \geq 0.$

2016

1. Solve the linear programming problem graphically 6
   Maximize and Minimize $z = 3x + 2y$
   Subject to $x + 3y \leq 60,$
   $x + y \geq 10,$
   $x \leq y,$
   and $x \geq 0, \quad y \geq 0.$
   OR

2. Maximize and Minimize $z = 5x + 7y$
   subject to $x + y \leq 4$,
   $3x + 8y \leq 24$,
   $10x + 7y \leq 35$
   $x \geq 0, \quad y \geq 0.$

2015

1. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's tome in its making, while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time. If the profit on racket and on a bat is Rs. 20 and Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity. 6

OR

2. Maximize and Minimize $z = x + 2y$
   subject to $x + 2y \geq 100$,
   $2x - y \leq 0$,
   $2x + y \leq 200$
   $x \geq 0, \quad y \geq 0.$

Chapter 13 *Probability* Marks(10)

2019

1. A natural number is selected at random from the set $A = \{x : 1 \leq x \leq 50\}$. Find the probability such that the number satisfies the inequation $x^2 - 13x \leq 30$. 4
2. Two numbers are selected at random from a set of first 90 natural numbers. Find the probability that the product of randomly selected numbers is divisible by 3. 6

   OR

3. In a 3x3 matrix, entries a_{ij} are selected randomly from the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ with replacement where each element a_{ij} is a three digit number. Find the probability that each element in each row is divisible by 15.

2018

1. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once. 4
   OR
2. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.
3. Find the variance of the number obtained on a throw of an unbiased die. 6

   OR

4. A and B throw a die alternatively till one of them gets a '6'and wins the game. Find their respective probabilities of winning, if A starts first.

2017

1. Two dice are thrown. Find the probability that the sum of the numbers coming up on them is 9, if it is known that the number 5 always occurs on the first die. 4

   OR

2. A bag contains 2 red and 4 black balls, another bag contains 3 red and 3 black balls. One of the two bags is selected at random and a ball is drawn from a bag which is found to be red. Find the probability that the ball is drawn from the first bag.
3. Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the mean, and variance of the number of Queens. 6

   OR

4. Six coins are tossed simultaneously. Find the probability of getting (i) 3 heads. (ii). no head, (iii). At least one head.

2016

1. Two cards are drawn successively, without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces. 4

   OR

2. A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box.
3. The sum and product of mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution. 6

   OR

4. If a fair coin is tossed 10 times, find the probability of getting (i). exactly six heads. (ii). at least six heads. (iii). At most six heads. 2+2+2=6

2015

1. Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls, given that (i). the youngest is a girl, (ii). at least one is a girl? 4

   OR

2. The probability of a shooter hitting a target is ¾. How many minimum number of times must he / she fire so that the probability of hitting the target at least once is more than 0.99 ?
3. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. 6

   OR

4. In a 20-question true-false examination, suppose a student tosses a fair coin to determine his answer to each question. If the coin falls head, he answers ‘true’. If it falls tail, he answers false. Find the probability that he answers at least 12 questions as true.